



## Development of similitude laws for inflatable structures under wind loading

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**ABSTRACT:** Representing wind loads on a rigid structure, at reduced scale in a boundary layer wind tunnel, is very common. But wind loads on a deformable structure, as inflatable ones, is quite a challenge to reproduce at a scale 1/100 to 1/500 mainly on the side of the weight of the fabrics and the stiffness of the structure itself. Inflatable structures are going to be more and more popular, due to their light weight, no foundations, fast and easy erection and finally, full recycling ability. Because large inflatable structures are used today for hosting temporary events, shelter airplanes or food reserves, protect people and goods from weather injuries, there is a strong need for the assessment of their behavior in strong wind conditions. This paper aims at showing a theoretical and experimental work to establish similarity laws to be used in the downscaling process of inflatable buildings for studying them in boundary layer wind tunnel. The first part of the study concerns mechanical modeling of inflatable structures to which a downscaling process is applied. After applying the Vashy-Buckingham theorem, three reduced-scale models size are manufactured to fulfill tests in a wind-tunnel to characterize the similarity conditions. This work conclusion reveals that after some adjustments it is possible to obtain the same behavior for two different size models with a scale ratio of 3.

**KEY WORDS:** Inflatable structures, wind loading, wind-tunnel, similitude.

### 1 INTRODUCTION

Inflatable structures are more and more used in Civil Engineering because they are low cost, light weight and more ecological compared to other equivalent structures. They are today sheltering public and/or equipment; therefore it is necessary to improve the knowledge of their behaviour. More, the European Union set a work group (CEN T6250 WG5) to establish design recommendations for tensored membranes which would include inflatable structures (Eurocodes). Some experimental tests need to be conducted to validate the models mentioned in the future regulations. This topic is also addressed in COST action TU1303 named “Novel structural skins: Improving sustainability and efficiency through new structural textile materials and design” which began in November 2013.

Inflatable structures might be very large so they cannot be tested at full scale in wind-tunnel for the assessment of their behaviour under wind loads. The question of the suitability of building scaled models is addressed here, in view of BLWT testing. The present paper introduces the development of similitude laws for inflatable structures and their experimental validation. It is shown that developing similitude relation is achievable on simple problems when the analytical ruling equations are known. For other cases, this paper introduces a new similitude model for inflatable structures based on Vashy-Buckingham Theory. The first step consists in applying the Vashy-Buckingham theorem to identify the dimensionless numbers that must be equal to ensure the same behaviour between two models at different scales. A numerical study on a simple inflatable beam problem is then carried out to validate the dimensionless number obtained before. An experimental work is then fulfilled to confirm the possibility of using reduced scale models of a complex shape in wind tunnel. Experimentation results highlight the similarity of the behaviour and the possibility to match the result from two models at different scales. But this conclusion is obtained after some modification in the similitude model that must be characterized analytically. It also concludes on the difficulty of working with textiles which are strongly non-linear materials.

## 2 MODELS FOR INFLATABLE STRUCTURES

### 2.1 Analytical expression for inflatable beams

Dealing with simple structures such as beam, several authors have derived analytical expressions for the load-deflection curve. Comer and Levy [1] first worked on an isotropic beam under Bernoulli's beam theory hypothesis. They have described the beam behavior before the apparition of the first wrinkle. They have also determined the wrinkling moment for the beam. The works from Comer and Levy have then been improved by Fichter [2] considering Timoshenko kinematics; this hypothesis is important for thin-walled structure where the shear effects can be important. Le Van and Wielgosz [3] then improved Fichter [2] results. Thomas [4] makes Le Van and Wielgosz [3] more accurate with a focus on inflation step. Le Van and Wielgosz [3] has finally got richer with Nguyen et al. [5] works who considered the orthotropic behavior and the inflation step effects for the bending study. They also propose beam formulas. Here is an example for a cantilever beam.

$$V(X) = \frac{F}{\left(E_l + \frac{P}{S}\right)I} \left( L \frac{X^2}{2} - \frac{X^3}{6} \right) + \frac{FX}{P + kG_{lt}S} \quad (1)$$

Where  $V$  is the deflection;  $F$  is the force;  $L$  is the beam length;  $S$  is the cross-section area;  $I$  is the second moment of area;  $P$  is the pressure contribution in stiffness;  $E_l$  is the longitudinal Young's Modulus;  $G_{lt}$  is the shear modulus and  $k$  the shear cross-section coefficient.

### 2.2 Numerical methods for inflatable structures

Some authors developed special finite elements for modeling inflatable beams. In this study the 2-nodes beam finite element from Thomas [6] is used. The lab also has its own 3D membrane code to simulate inflatable beams (or more complex structures) behavior. For showing the quality of various models a basic bending test was fulfilled (

Figure 1). An inflatable beam made of orthotropic technical textile F302 from Ferrari was installed on two rollers 4m spaced. A rope was used to apply a force at the middle of the beam to make it bend. The diameter of the beam was 0.206m and the inflation pressure 25 kPa. The experimental load-displacement curve was built using a camera and Regressi software. It only supplies this evolution at the middle of the beam. A good accuracy was found at the beginning and a small gap appeared as long as the strength increased. This gap was partly explained by the properties of chosen material which are given for the textile. The beam properties differs a few from the textile one [5]. This coefficient can be adjusted with experimental result but the aim here was only to show the ability of the model. The correlation between the finite elements methods and the theoretical curve was made by Nguyen [5].

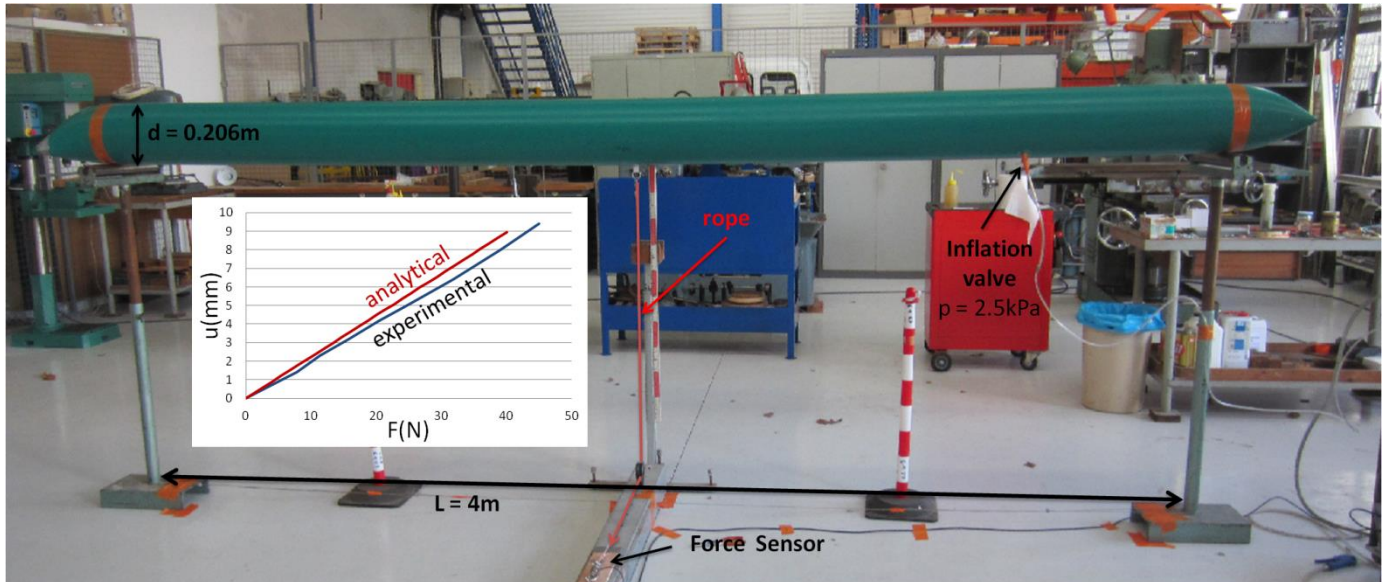


Figure 1. Experimental set-up for bending test and results.

### 2.3 Models limitation

As mentioned before most of the models are efficient before the wrinkle apparition, especially the analytical expression ones and the beam finite element ones. Having an analytical expression is ideal to obtain the similitude profile but it is only available

on simple beam so finite element modeling must be used for beam assembly or more complex beam-based structures. The 3D membrane element is essential for other geometries. Moreover, it is hard to take the fluid-structure interaction aspect into account especially on reduced-scale models especially since the wind load distribution is not available due to continuous deformation of the structure : how to build a smaller model?

### 3 SIMILITUDE LAWS FOR INFLATABLES

Some studies concerning the textile aerodynamic behavior have already been conducted especially for sports [7][8] or for spatial application [9]. However, only few works exist about similitude. A report from CSTB proposed a first analytical development from the equilibrium equations of a single-walled-and-pressurized membrane [10]. Tryggvason and Isyumov [11] presented some similarity requirements for this kind of membrane. Tryggvason [12] was also considering an aeroelastic modeling.

#### 3.1 First approach with inflatable beams

When the theoretical load-deflection curves are available for inflated beams, it is easy to rearrange the analytical expression to make them dimensionless and to define the non-dimensional numbers ruling this expression. Those numbers must be the same from one configuration to another for the similitude to be applied. The equation given by Nguyen for an inflatable cantilever beam is rewritten with dimensionless numbers as:

$$\frac{V(X)}{L} = \frac{FL^2}{\left(E_t + \frac{P}{S}\right)I} \left( \frac{1}{2} \left( \frac{X}{L} \right)^2 - \frac{1}{6} \left( \frac{X}{L} \right)^3 \right) + \frac{F}{P + kG_{it}S} \frac{X}{L} \quad (2)$$

The two dimensionless numbers that must be the same from one problem to another are then:

$$\Pi_1 = \frac{FL^2}{\left(E_t + \frac{P}{S}\right)I}; \quad \Pi_2 = \frac{F}{P + kG_{it}S} \quad (3)$$

This development is very classical to go from one model to another. However, it is restricted by the knowledge of the analytical model. For more complex problems another way to build the reduced-scale models must be considered.

#### 3.2 Second approach for more complex geometries

The Vashy-Buckingham theorem settles the similitude theory. It allows to build a finite number of non-dimensional groups ruling the physical problem. The method is to first write all the variables constituting the physical problem and then express them in the international unit system (Table 1). The dimensional analysis gives the dimensionless numbers.

Symbol	Signification	Expression in SI
v	Fluid velocity	m/s
$\mu$	Dynamic viscosity	kg.s/m
$\rho$	Fluid density	kg/m <sup>3</sup>
L	Length	M
F	Force	kg.m/s <sup>2</sup>
$E_t$	Fabrics Young's modulus in thickness	kg/s <sup>2</sup>
$G_{it}$	Fabrics Shear modulus in thickness	kg/s <sup>2</sup>
p	Inflation pressure	kg/(m.s <sup>2</sup> )
$\rho_f$	Fabrics density	kg/m <sup>3</sup>

Table 1. Expressions in SI units the physical problem variables.

From the theorem, the following groups are obtained:

$$\Pi_1 = \frac{\mu}{\rho L} = \frac{1}{Re}; \quad \Pi_2 = \frac{F}{\rho v^2 L^2}; \quad \Pi_3 = \frac{E_t t}{\rho v^2 L}; \quad \Pi_4 = \frac{G_{it} t}{\rho v^2 L}; \quad \Pi_5 = \frac{p}{\rho v^2}; \quad \Pi_6 = \frac{\rho_f}{\rho}; \quad (4)$$

Those numbers allow the scale effects identification for each variable:

Scale = 1/N	Considering Re to stay the same	Considering the same material
$E_{lt}, G_{lt}$	N	1
$\rho_f$	1	1
L	1/N	1/N
F	1	1/N
p	$N^2$	N
Re	1	$N^{0.5}$
v	N	$N^{0.5}$

Table 2. Scale ratios between the variables when working at the same Reynolds or with the same material.

Scale ratios at the same Reynolds number (Table 2) highlight the difficulty of keeping the same flow conditions. The realization of such a model would need a very stiff material and a very big air-compressor to apply the required inflation pressure. The velocity is also quickly increasing with size reduction. The final objective is to be able to build models reduced at scale 1/200 to test them in BLWT. Applying this ratio would ask for supersonic wind speeds and very strong fabrics. Scale effects can be alleviated by keeping the same material. In this case the Reynolds similitude is not satisfied. Before carrying experimental works in the wind-tunnel, this scale effects are applied on numerical cases to check the behavior on a simple beam (Figure 2).

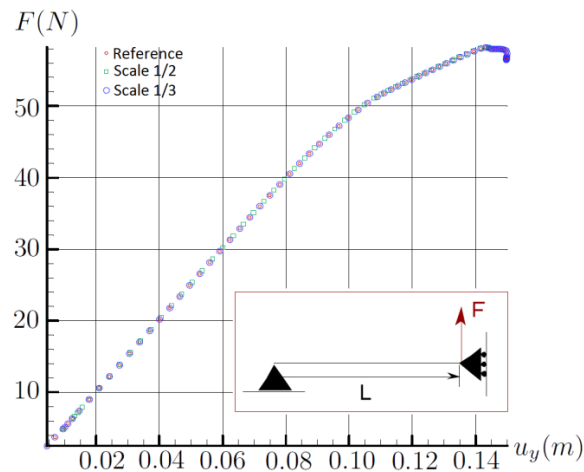


Figure 2. Numerical validation of dimensionless numbers.

## 4 EXPERIMENTATION IN WIND-TUNNEL

### 4.1 Presentation and objectives

After dimensionless numbers have been validated numerically, tests were carried out to state on the ability to put the method into practice. To do so, 3 models were built, using in the same fabrics which is Ferrari F302 technical textile, in 3 different sizes. The models were designed with respect to scale effects limitation on inflation pressure. In order to evaluate the influence of each parameter two additional models were made in a different textile that is Terraza 550884. The models shape and geometry are chosen from an existing structure, at a scale 3 times larger than the largest wind tunnel model SG1, but too large, 10m x 16m, for wind tunnel testing. The aim is to reproduce the wrinkle and the ruin phenomenon at reduced-scale.

### 4.2 Models dimensions

The models manufactured for the test to be fulfilled are the same shape. The reference model (SG1) is a half-cylinder formed by a succession of 10 inflatable arches. Other models represent the reference model at two different scales (1/3 for SG2 and 1/9 for SG3). Models dimensions are given by Table 3 below.

	Model	Scale	Arches	D[m]	L[m]	d[m]	$D_{SG1}/D$	$L_{SG1}/L$	$d_{SG1}/d$
	SG1	1	10	5.30	3.40	0.34	1.0	1.0	1.0
	SG2	1/3	10	1.80	1.13	0.11	2.9	3.0	3.1
	SG3	1/9	10	0.60	0.44	0.04	8.8	7.7	8.0

Table 3. Models dimensions.

Technical limitation to make the smaller size models requires the use of sewing. The models then need to be airtight, therefore a liquid was poured inside to plug the needle holes. This liquid is intended for Zodiac boats. All the models built are presented below in Table 4, the maximal inflation pressure is obtained experimentally:





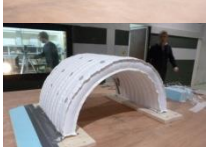
Model	Name	Size	Fabrics	Pressure max (kPa)
	SG1-1	SG1	Ferrarri F302	1.25-1.38
	SG2-1	SG2	Ferrarri F302	up to 4.14
	SG2-2	SG2	Terraza 880884	up to 2.5
	SG3-1	SG3	Ferrarri F302	up to 12.42
	SG3-2	SG3	Terraza 880884	up to 6

Table 4. Panorama of the built models.

#### 4.3 Experimental set-up

The models are successively placed in the Jules Verne wind-tunnel high-speed area. The tunnel there is 6m wide and 5m high. The wind speed possibly goes up to 80m/s. The biggest structure (SG1) which is 2.7m high was put on a wood frame which was directly fixed on the floor. The two other (SG2 and SG3) were attached on a 1.2m high table to break free from the boundary layer influence (Figure 3). The wood frame for SG1 and the table for SG2 and SG3 are on a rotating pan for testing several wind directions.



Figure 3. Models installation in wind-tunnel.

The question of anchorages is difficult to answer when dealing with inflatable structures. How to make sure that the anchorage are not going to damage the structure? The adopted solution is to sew several textile loops along the structure basis. Loops are



then pressed against the bearing support using screws and washers. SG2 and SG3 are screwed on the supporting table while SG1 is screwed on its wood-frame (Figure 4).



Figure 4. View of the fixing system for models.

Displacements measurements were done using 3D point optical tracker system (issued from a commercial Motion Capture system), which has a precision of 1mm over a 100m<sup>3</sup> volume at a recording rate up to 600 Hz. The 3D tracker system is based on infrared video-cameras which are following infrared-reflector targets previously stuck on the studied structure (figure 4). 3 others control stations are installed in addition with the motion capture system. The first one is dedicated to the inflation pressure regulation; the second one is occupied by the wind-tunnel operator and the last one supplies the external pressure field on the structure (Figure 5).

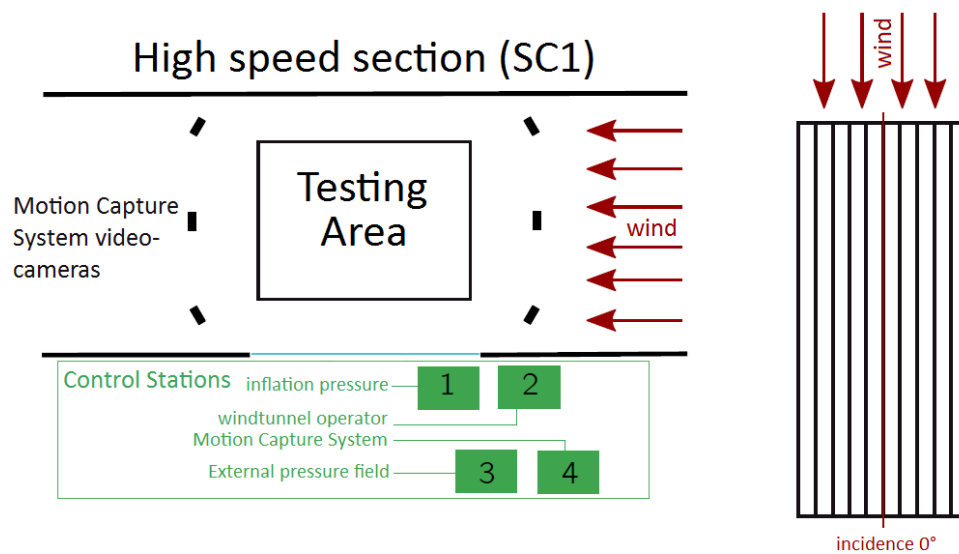


Figure 5. Overview of the experimental set-up.

The study on wind direction shows that the more interesting configuration is when the structure is placed parallel to the wind direction. Only those results will be discussed hereafter.

#### 4.4 Results

Model	Inflation pressure (kPa)	Windspeeds (m/s)
SG1-1	1.38	5, 10, 12, 15, 18
SG2-2	4.14	9, 17.5, 21, 26, 31
SG3-2	1.24	15, 30, 36, 45, 54

Table 5. Pressure and wind-speeds to be applied according to the similitude analysis.

The experimental protocol starts with the strict application of similitude laws using the same material (Table 5). The three models considered are SG1-1 plus SG2-1 and SG3-1. The pressure applied in SG1-1 was 1.38 kPa. The study is focused on displacement field. The aim is to reproduce the behavior of SG1-1 with the use of SG2-1 and SG2-3. The study begins with some general behavior aspect. The geometry considers 10 inflatable arches but the general overview of the loading movement concludes that each arch presents the same behavior (Figure 6). Only arch 7 is analyzed for the following. This displacement of various targets plotted in figure 6 was obtained during a pressure loss event under a constant wind.

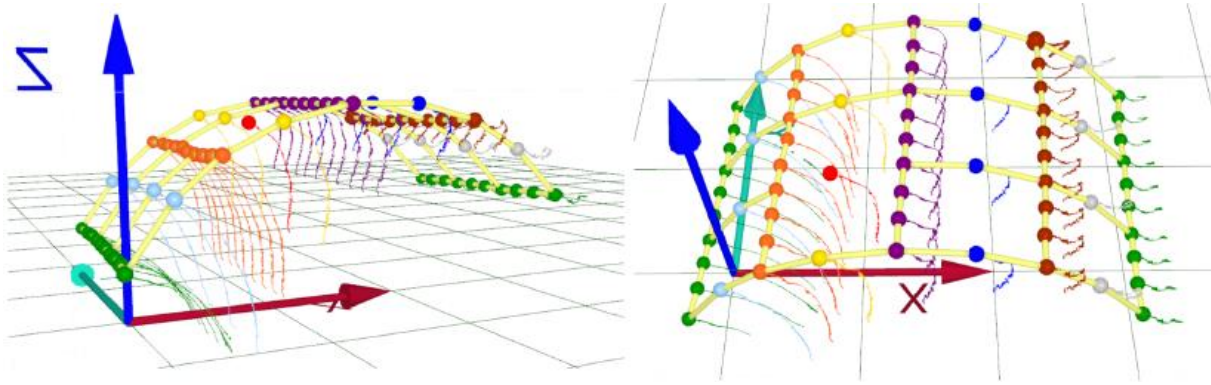


Figure 6. Global behavior overview for SG2 type structure during a pressure loss event.

Another result highlights the difference from the loading phase (when the wind is increasing in speed) and the relaxation phase (when the wind speed is decreasing). Figure 7 is built on SG2 type structure: the wind linearly ranges from 0 to 20 m/s then from 20 to 0 m/s. During the loading, the pathway seems to be very complex and non-linear. During the relaxation, points are choosing the easiest way to get the initial configuration back (Figure 7).

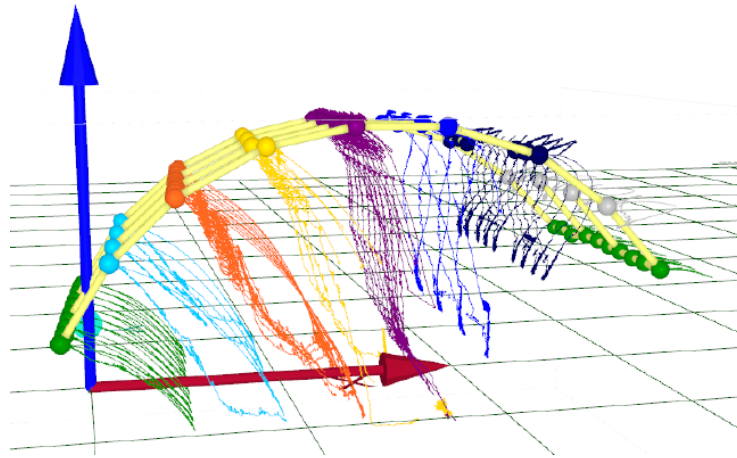


Figure 7. Points trajectories during a loading-unloading test for SG2 type structure.

The displacements of SG1-1 are expressed in norm:

$$\|\vec{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2} \quad (5)$$

Where  $u_x$  is the displacement component on  $x$  direction.

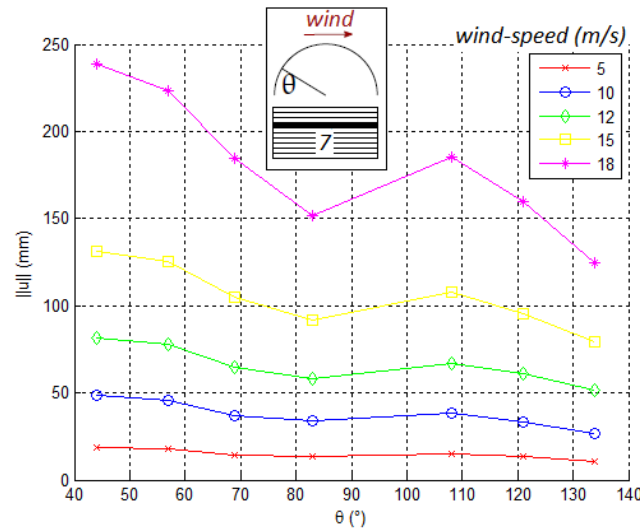


Figure 8. SG1-1 displacements for several wind-speeds.

The displacements from SG3-1 are very low and there is no structure deformation to be seen (Figure 9). The model seems to be too stiff. The theoretical ratio of 9 to get the displacement of SG1-1 (Figure 8) was not identified. Actually, the model can bear itself without any pressure inside. The displacements are more obvious on SG2-1 (Figure 10) but the theoretical coefficient of 3 is not confirmed by experimentation.

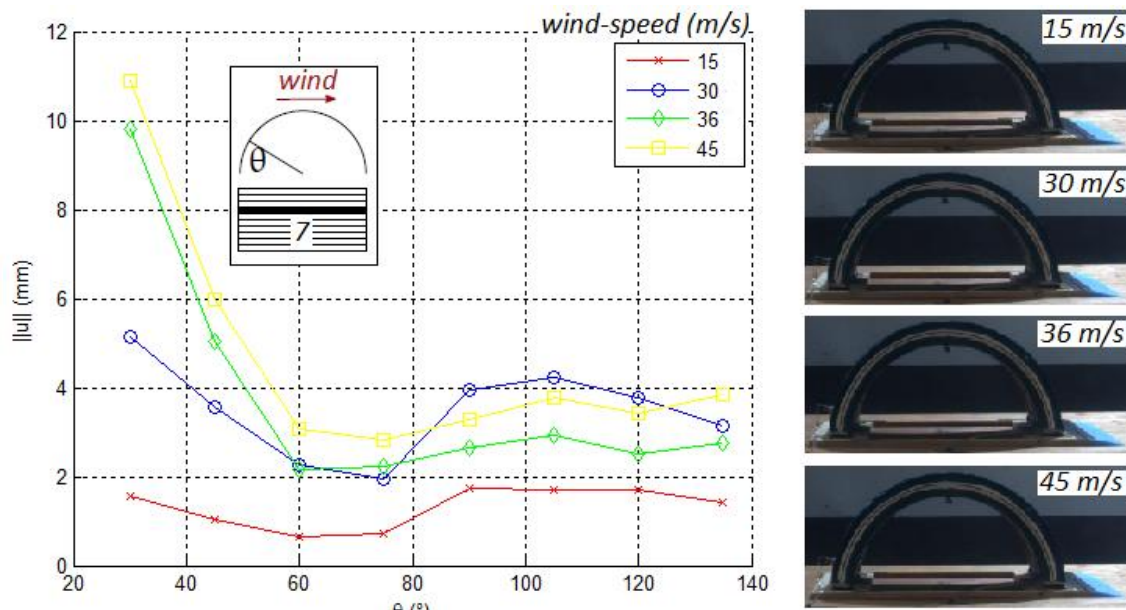


Figure 9. SG3-1 displacements for several wind-speeds.

Keeping the same material means that the textile thickness is not at scale. That can explain the over stiffness observed. Moreover, the ratio textile volume over total volume increases when the scale diminishes. The idea right here is to try to adjust the inflation pressure to find the anticipated displacement. This experiment is difficult to realize on SG3-1 because of the fabrics weight. In fact, a minimal pressure is needed to counterbalance gravity effects. As a result, there is not enough margin to envisage a consequent pressure diminution. Contrary to SG3-1, it is possible to adapt the inflation pressure in SG2-1. The displacement of SG1 are get with a transition factor of 3 on SG2-1 (Figure 11) for an inflation pressure of 3 kPa (instead of 4 kPa). A future analysis will be led to quantify this variation analytically.



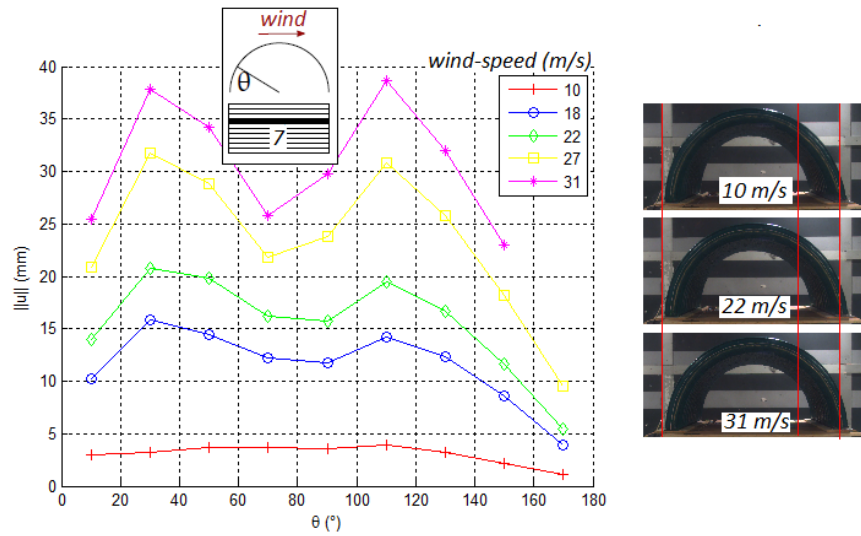


Figure 10. SG2-1 displacements for several wind-speeds.

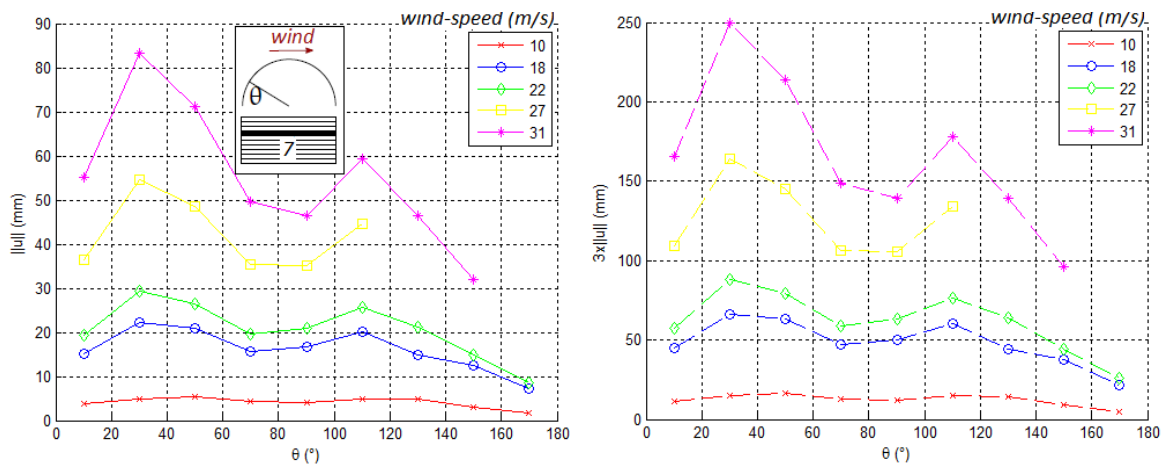


Figure 11. SG2-1 with modified inflation pressure displacements for several wind-speeds.

In this study, a change of material has also been considered but the results (Figure 12) are not showing a clear conclusion on the importance of the material. Because it is not possible to raise the pressure more than 2 kPa, no comparison between both material at the same pressure can be done.

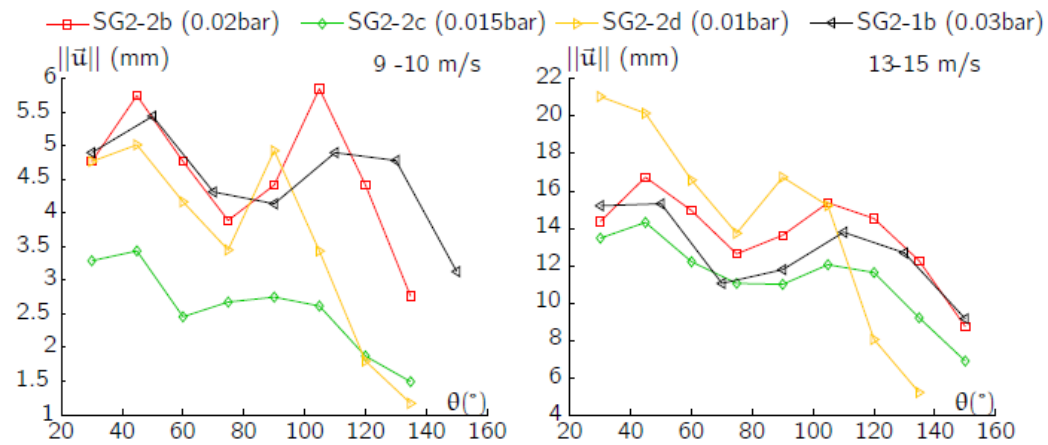


Figure 12. SG2-1 Comparison of displacements of the same size models using two different materials.

The final step is the measurement of external pressure field. This information is of primary interest to continue the building of the similitude model. Even if the field seems to be known because the structure shape is cylindrical, inflatable structures deal with large displacement. The key information is the geometry influence on the external pressure field. It can be seen on results (Figure 14) that on the initial configuration the  $C_p$  distribution is close to the typical distribution for cylinders. The detachment occurs near  $35-40^\circ$  and the values range between  $[-1.2; 0.8]$ . When the wind is stronger, the structure shape changes and the distribution is affected. The angle for the location of detachment point becomes more important and the  $C_p$  is translated but still ranges in the same interval. The nature of inflatable structures cannot admit the use of classical pressure taps: it is not allowed to drill such a structure. Special captors were built for this study: they were manufactured from existing flat cylindrical shape plastic elements with a 0.8mm hole drilled perpendicular to its upper surface (Figure 13).



Figure 13. A view of specially designed pressure captors.

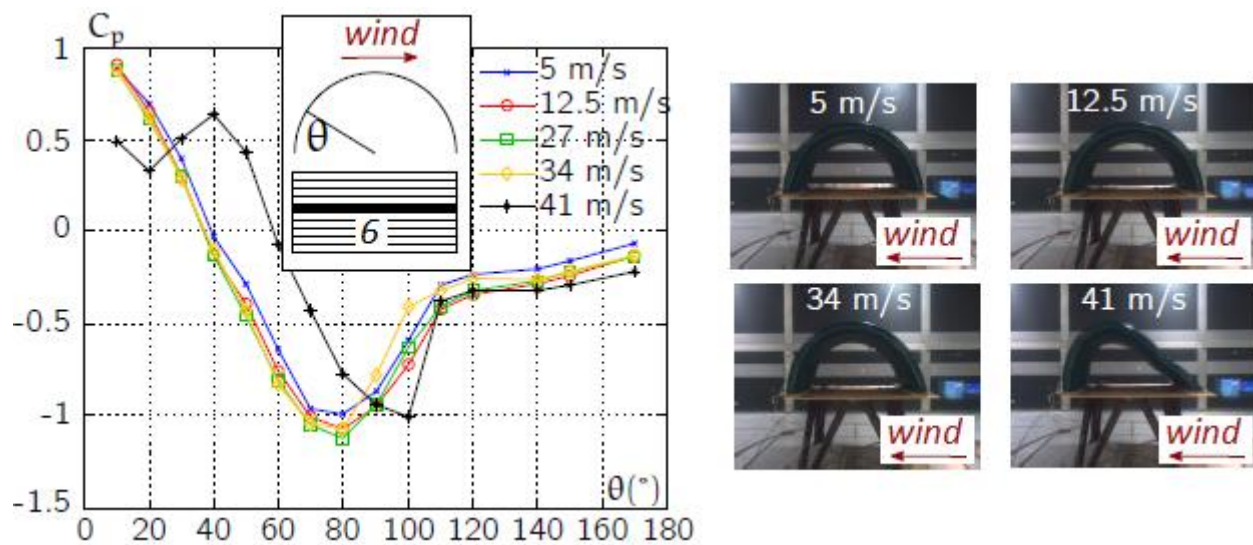


Figure 14. External pressure field distribution evolution while wind speed rising.

## 5 CONCLUSION

The study has shown the possibility of introducing a similitude law for simple inflatable structures, such as a simple beam, when analytical expressions are available. For more complex geometries the experimentation highlighted the reduced-scale model stiffness incompatibility to ensure a good behavior in comparison with the reference prototype. To correct the over-stiffness of small scale models some tracks have been proposed: correcting the inflation pressure in relation to the textile volume fraction drop or by using a lighter material. This solution needs to find new fabrics suppliers or fabrication process. Another

option is to reduce the number of arches on small models. This option would lead to limit the leaking air volume so the pressure inside should be higher. Some aspects of textile behavior such as creep or plasticity have also to be taken into account to represent the whole reaction of a real structure with models. As a conclusion, this study is supplying a solid basis for future experimentations on small scale models, the change due to materials and pressure has been observed. The external pressure field measurement is also primordial: it is possible to do numerical analysis to develop a stiffness model since the loading is known. All those results establish the first step on our way to establish practicable similitude laws for downscaling inflatable structures for BLWT testing

#### ACKNOWLEDGMENTS :

This study and the PhD thesis by Alexis Bloch from which it is issued were made possible by the full financial support of LIRGEC, the Ligerian Research network for Civil Engineering, funded by the Region of Pays de la Loire.

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